# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2023**

**MT 6603 – COMPLEX ANALYSIS**

Date: 15-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

**Part A**

**Answer ALL questions:** **(10 x 2 = 20)**

1. Find the absolute value of $\frac{(1+3i)(1-2i)}{3+4i}$.
2. Define harmonic function.
3. Find the radius of convergence of the series $\sum_{n=1}^{\infty}\frac{z^n}{n}$.
4. Using Cauchy integral formula, evaluate $\int_C \frac{e^z}{z}$ where $C$ is the unit circle $|z| = 1$.
5. State Morera's theorem.
6. Write Maclaurin series expansion of $sin\, z$.
7. Define removable and essential singularities.
8. State Argument principle.
9. Define critical point.
10. Define a bilinear transformation.

**Part-B**

**Answer any FIVE questions:** **(5 x 8 = 40)**

11. Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied at the origin.
12. Prove that $u(x, y) = y^3 - 3x^2y$ is harmonic and find it's harmonic conjugate.
13. State and prove Liouville's theorem.
14. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ in a Laurent's series in (i) $1 < |z| < 2$ and (ii) $|z| > 2$.
15. State and prove Cauchy's residue theorem.
16. Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2}\, dz$ where $C$ is $|z| = 3$.
17. Show that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.
18. Find the bilinear transformation which maps the points $z_1 = 2,\ z_2 = i,\ z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$.

**Part C**

**Answer any TWO questions:** **(2 x 20 = 40)**

19. State and prove the necessary and sufficient condition for differentiability of a complex valued function.

20. (a) State and prove Cauchy integral formula.
    (b) Using contour integration evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\,sin\theta}$. **(12+8)**

21. (a) State and Prove Taylor's theorem.
    (b) Show that $\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ when $|z-2| < 2$. **(12+8)**

22. (a) Show that any bilinear transformation which maps the unit circle $|z| = 1$ onto $|w| = 1$ can be written in the form $w = e^{i\lambda}\left(\frac{z-\alpha}{\bar{\alpha}z-1}\right)$ where $\lambda$ is real.
    (b) State and prove Rouche's theorem.

**(10 + 10)**

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